The Optimal Level of Liquid Assets: An Empirical Test

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■ The amount and composition of liquid assets (defined as cash plus marketable securities) that a firm should hold is of interest to practitioners and academicians alike. Virtually every textbook on corporate finance analyzes the cost-benefit tradeoff of holding liquid assets and argues that this tradeoff dictates an optimal level of liquid assets. The estimation of these liquidity needs has been analyzed by Stone [8] and Gitman [2].[1]

This paper's concern is with testing empirically for the existence of an optimal level of liquid assets for firms in a given industry. Such a test can provide indirect evidence on the textbook theories of optimal liquidity. For financial analysts, it also can allow for more specific interpretation of observed liquidity when performing comparative ratio analyses.

Section I briefly summarizes the traditional argument for the existence of an optimal level of liquid assets. The empirical model is developed and hypothesis tests are discussed in Section II. The sample selection and test results are discussed in Section III.

I. Optimal Level of Investment in Liquid Assets[2]

Apart from compensating balance requirements imposed by banks, firms are thought to hold liquid assets for the transaction and precautionary reasons recognized by Keynes [5]. The transaction motive arises from the nonsynchronization of cash inflows and outflows. The precautionary motive arises from the possibility of unexpected cash needs. Inadequate holdings of liquid assets expose the firm to "shortage" costs, such as missed discounts on trade credit, higher trans-

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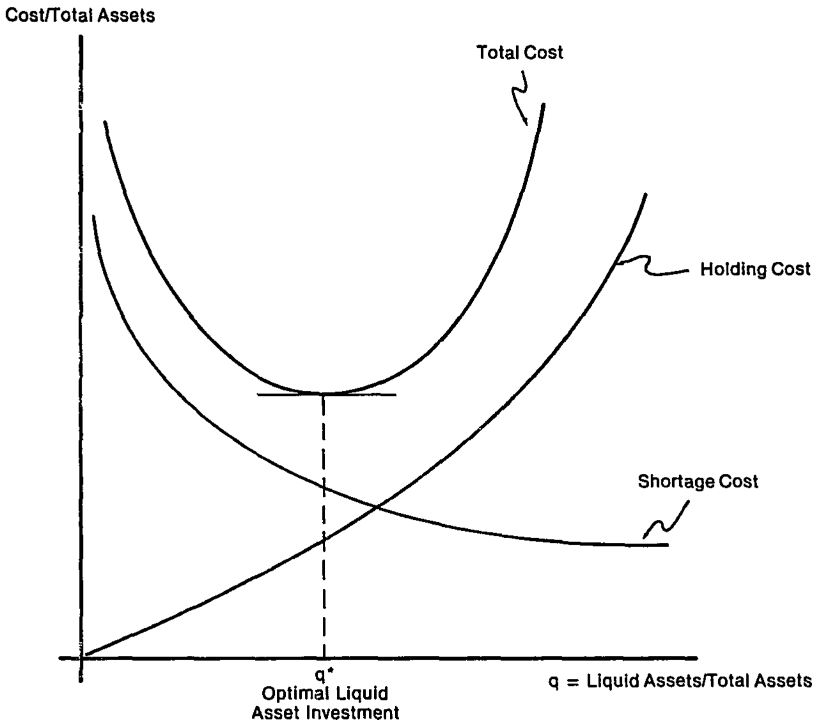
[1]The division of liquid assets between cash and marketable securities has been discussed by Baumol [1], under certainty, and Miller and Orr [6], under uncertainty. For a more complete list of references and a historical review of the general area see Gitman, Moses, and White [3].

[2]Similar presentations of the argument presented in this section are given by Higgins [4, pp. 322–323], and Moyer, McGuigan, and Kretlow [7, pp. 466–468] as well as other financial management texts.

Exhibit 1. Cost of Investment in Liquid Assets



action costs in converting illiquid assets, higher interest rates encountered with quickly negotiated loans, possible lower credit ratings, and the expected costs of insolvency. These shortage costs decline as liquid asset balances increase.

On the other hand, liquid assets impose "holding" costs on the firm. These arise from forgone opportunities to invest in less liquid but more productive assets. For example, liquid assets are subject to double taxation when held by firms, and purely from a tax standpoint, it may thus be more advantageous for investors to hold liquid assets directly. These holding costs increase as the firm's investment in liquid assets increases, and the firm thus is faced with the classical tradeoff between liquidity and profitability.

The combination of shortage and holding costs produces a U-shaped total cost curve, as depicted in Exhibit 1. The level of liquid assets corresponding to the minimum point of this total cost curve represents the optimum.

The level of liquid asset holdings will also be reflected in the returns to shareholders. As liquid asset holdings increase toward the optimum, shareholder returns should increase, but as liquid asset holdings increase beyond the optimum, shareholder returns should decrease.

II. Methodology and Hypotheses

The capital asset pricing model (CAPM) specifies the *ex ante* or expected rates of return to shareholders as a function of the risk-free rate, the expected return on the market, and the firm's systematic risk level, or beta. However, in *ex post* terms, company-specific factors, not fully reflected in systematic risk levels, give rise to abnormal returns, both positive and negative. Since a firm's liquid asset investment decision and its associated costs are to some degree company-specific, it is one such factor that could result in abnormal returns. To capture this effect, a function of liquid asset levels is added to an *ex post* empirical counterpart of the CAPM:

$$r_j = r_f + (r_m - r_f)\beta_j + \Theta(q_j) + e_j, \qquad (1)$$

where r_j represents the returns on company j, r_f the risk-free rate, r_m the return on the market portfolio, β_j the systematic risk level for company j, $\Theta(\cdot)$ a function that is concave downward, q_j the level of j's investment in liquid assets (relative to total assets), and e a random error term.

Many funtional forms would reflect a concave relationship between returns and liquid assets levels, one of which is a quadratic. Taking $\Theta(\cdot)$ as a quadratic, and moving the CAPM-related components to the left-hand side, the cross-sectional regression equation corresponding to Equation (1) is

$$r_j - r_f - (r_m - r_f)\beta_j = \alpha_0 + \alpha_1 q_j + \alpha_2 q_j^2 + e_j, \quad (j = 1, \ldots, J) \qquad (2)$$

where the α's are regression coefficients.[3] For this cross-sectional regression to be justified, the parameters (α's) in the function of liquid assets must be constant across firms in the sample. This condition is probably not satisfied for firms from different industries, since the factors (*i.e.*, trade credit terms, nature of illiquid assets, bankruptcy costs, *etc.*) resulting in shortage and holding costs would differ across industries. However, within a given industry the condition should be approximately satisfied.[4]

The optimal level of liquid assets, as given by the first order condition of Equation (2), is

$$q_j = -\alpha_1/2\alpha_2. \qquad (3)$$

Obviously, the exact optimum will vary across firms, as is evident from the disturbance term. This random element is assumed to arise from omitted variables, such as management expertise.

In addition, it should be recognized that the costs and rewards of holding liquid assets vary over time. Thus, it is possible that variations in the optimum over time might result in the majority of firms operating with liquid asset balances either above or below the optimal level during a given period. Therefore, estimation of Equation (2) based on a single cross-section could yield misleading results, in that, for one particular period, the data might only reflect a positive or a negative relationship between excess returns and liquid asset levels. Just such a phenomenon was observed by Townsend [11], who found that a simple linear cross-sectional relationship between liquidity and price performance switched from negative to positive during periods of tight money.

The dynamic nature of liquid asset management can be incorporated by combining a number of cross-sections over time. If the regression coefficients were fixed across time, the data could simply be pooled. However, as mentioned, the reward (α_1) and penalty (α_2) coefficients probably vary over time. This could result from differing interest rates on marketable securities, changes in market-wide liquidity, and other factors affecting the expected cost of illiquidity. Thus, expression (2) can be rewritten so as to encompass different periods:

$$r_{jt} - r_{ft} - (r_{mt} - r_{ft})\beta_{jt} = \alpha_{0t} + \alpha_{1t} q_{jt} + \alpha_{2t} q_{jt}^2 + e_{jt}, \quad (t = 1, \ldots, T) \qquad (4)$$

Taking the variation in the regression coefficients as random, the coefficients in each time period t are specified as

$$\alpha_{kt} = \alpha_k + Z_{kt}, \qquad (k = 0, 1, 2) \qquad (5)$$

where α_k is the mean or stationary portion of the k^{th} coefficient and Z_{kt} is the portion that varies randomly about zero over time. The specification given by Equations (4) and (5) can be estimated with Swamy's [9] random coefficient regression procedure, using a time-series of cross-sectional data sets. Thus, the relationship, although originally stated as a cross-sectional model, is estimated so as to exploit both cross-sectional and time-series variation in the data. The appropriateness of the random coefficient regression procedure can be tested using the homogeneity statistic, which was shown by Zellner [12] to follow an F distribution with $(T - 1)K$ and $T(J - K)$ degrees of freedom. If the hypothesized relationship holds between shareholder returns and liquid assets, one would expect α_1 to

[3]The use of q and q squared might suggest multicollinearity. However, if the distribution of q is symmetric it is uncorrelated with q squared, Thiel [10, p. 550]. Also, the effect on the hypotheses tests of multicollinearity would be to bias the tests against finding support for the existence of an optimal level.

[4]To the extent that the parameters are company specific within an industry, resulting in heteroscedastic disturbances, the least squares estimate is less efficient, but still an unbiased estimate.

be positive and α_2 negative. The null and alternative hypotheses are stated as follows:

Null	Alternative
$H_{10}: \alpha_1 \leq 0$	$H_{1a}: \alpha_1 > 0$
$H_{20}: \alpha_2 \geq 0$	$H_{2a}: \alpha_2 < 0$

Thus, rejection of the null hypotheses in favor of the alternatives would support the existence of an optimal level of liquid assets.

III. Data and Results

The 1981 COMPUSTAT Industrial and 1982 CRSP Monthly Return files were screened to obtain the industries providing the largest available samples. The data were screened to ensure that all firms were within a given industry (four digit SIC code); complete data were available; all firms had December fiscal year ends; and no firms were included from the financial institution or utility industries. The data collected were as follows: ending cash and market securities balances; ending total assets; the corresponding annual returns including dividends; and 60 months of returns preceeding each annual cross-section in order to compute the series of systematic risk levels for each firm. The resulting industries are chemical and allied products (SIC 2800), and petroleum refining (SIC 2911) for the common ten-year period 1968 through 1977.[5]

The CRSP value weighted market index including dividends was used in computing the annual market return (r_{mt}) and in estimating betas (β_{jt}) for each of the cross-sections of Equation (4). The risk-free rate (r_{ft}) was taken as the return on one-year U.S. Treasury bills computed from quotations given in *The Wall Street Journal*.

For the ten cross-sections from the chemical products industry, the value of the homogeneity statistic is 12.40, which is statistically significant at well above the 1% level for 27 and 110 degrees of freedom. The value of the statistic for the petroleum refining industry is 6.34, which is likewise statistically significant at well above the 1% level for 27 and 150 degrees of freedom.[6] Thus, for each industry the hypothesis that the coefficients are the same across time is rejected in favor of the hypothesis that they vary randomly. The random coefficient regression procedure was applied to the data, and the results of the estimation are presented in Exhibit 2.

Exhibit 2. Random Coefficient Regression Estimates

	α_0	α_1	α_2
	Chemical Products Industry (SIC 2800)		
Coefficient	−0.28	11.41*	−94.30*
(t-value)	(−1.92)	(3.57)	(−3.02)
	Petroleum Refining Industry (SIC 2911)		
Coefficient	−0.10	5.03*	−25.41*
(t-value)	(−1.06)	(2.41)	(−2.07)
	"Electric and Electronic Equipment & Components Mft." (SIC 3600–3699)		
Coefficient	−0.05	2.63*	−14.11*
(t-value)	(−0.47)	(2.33)	(−3.17)
	"Transportation Vehicle, Parts, & Equipment Mft." (SIC 3700–3799)		
Coefficient	−0.15	5.64	−27.25
(t-value)	(−0.96)	(1.32)	(−1.14)

*Statistically significant at the .025 level based on a one-tail test.

The estimated slope coefficients support the hypothesis of an optimal level of investment in liquid assets. The null hypotheses that α_1 is less than or equal to zero and that α_2 is greater than or equal to zero can be rejected at the 2.5% level for the chemical products industry. The null hypotheses can also be rejected at the 2.5% level for the petroleum refining industry.

In an attempt to broaden the range of data, the same estimation procedure was applied to two additional industry groups, selected using the less restrictive two digit SIC codes. These two additional samples consist of what might be loosely classified as the "electric and electronic components and equipment manufacturing" industry (SIC 3600–3699) and the "transportation vehicle, parts, and equipment manufacturing" industry (SIC 3700–3799).[7] The estimation results for the two more leniently defined industries are also given in Exhibit 2. Not surprisingly, the results offer mixed support for the hypothesis of an optimal level of investment in liquid assets. Although both samples produce α_1 and α_2 coefficients whose signs are consistent with the alternative hypotheses, only those for the "electric and electronic components and equipment manufactur-

[5]The petroleum refining industry provides 20 firms and the chemical products industry provides 16 firms. A list of firms is available from the author upon request.

[6]In examining the residuals from the OLS cross-sectional regressions two firms in each industry were found to produce large residuals in approximately 50% of the regressions and were eliminated as outliers. Thus, the sample sizes are reduced to 18 for the petroleum refining industry and 14 for the chemical products industry.

[7]The sample based on SIC codes from 3600 to 3699 contains 25 firms and the one based on SIC codes from 3700 to 3799 contains 24 firms. A list of firms is available from the author upon request.

ing" sample are statistically significant. However, one should not expect the tests to work as well with less homogeneous samples.

The results for the chemical products and petroleum refining industries provide an interesting opportunity to consider the practice of using industry averages as the norm or standard against which individual company ratios are compared. Based on the first order condition given in Equation (3) and the estimated coefficients, the optimal level of liquid assets for the chemical products industry is, on average over the ten-year period, 6.0% of total assets. The industry average level over this period is 5.9%. Thus, for the chemical products industry from 1968 through 1977, the optimal liquid assets level appears to have been virtually equal to the industry average. For the petroleum refining industry the optimal level of liquid assets, based on Equation (3) and the estimated coefficients, is 9.9%. The corresponding average level of liquid assets for the period 1968 through 1977 is 7.0%. Thus, for the petroleum refining industry the optimal liquid assets level appears to have been slightly above the industry average for the period. Since the larger absolute size of the coefficients, α_1 and α_2, suggests a more sharply defined optimum for the chemical products industry, it may not be surprising that chemical products firms appear to have managed their liquid assets more closely.

IV. Conclusion

In summary, this study supports the existence of an optimal level of investment in liquid assets which varies over time. As the relative amount of liquid assets is increased, returns initially increase because of significant reductions in shortage costs. Beyond some optimal level, returns begin to decline as mounting holding costs exceed the reduction in shortage costs. Thus the arguments for optimal liquid asset levels contained in financial management texts appear to have some empirical validity.

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